



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The volume of the tetrahedron found by the points is

$$\frac{1}{3!} \begin{vmatrix} x_1, & y_1, & z_1, & 1 \\ x_2, & y_2, & z_2, & 1 \\ x_3, & y_3, & z_3, & 1 \\ x_4, & y_4, & z_4, & 1 \end{vmatrix}$$

or substituting the values of x_1, y_1, z_1 , etc., we have

$$\begin{aligned} \text{Volume} &= \frac{1}{6} \begin{vmatrix} A_1, & B_1, & C_1, & D_1 \\ A_2, & B_2, & C_2, & D_2 \\ A_3, & B_3, & C_3, & D_3 \\ A_4, & B_4, & C_4, & D_4 \end{vmatrix} \div D_1 D_2 D_3 D_4 \\ &= \frac{1}{6} \Delta^3 \div \Delta_1 \Delta_2 \Delta_3 \Delta_4 = F. \end{aligned}$$

Also solved by *G. B. M. ZERR, WALTER H. DRANE*, and the *PROPCSER*.
Professor Carter asks: What does F represent when n is greater than 4?

CALCULUS.

97. Proposed by *ARTEMAS MARTIN, A.M., Ph.D., LL.D.*, United States Coast and Geodetic Survey Office, Washington, D. C.

An auger hole, radius r , is bored through a prolate spheroid; the axis of the auger passing through the center, perpendicular to the major axis. Find the volume removed.

Solution by *G. B. M. ZERR, A. M., Ph. D.*, Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$, be the equation to the prolate spheroid.

$x^2 + y^2 = r^2$, the equation to the cylinder.

$$\begin{aligned} \therefore V &= 8/a \int_0^r \int_0^{\sqrt{r^2 - x^2}} \sqrt{[b^2(a^2 - x^2) - a^2 y^2]} dx dy \\ &= 4/a \int_0^r \{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \} dx \\ &\quad + \frac{4b^2}{a^2} \int_0^r (a^2 - x^2) \sin^{-1} \left[\frac{a}{b} \sqrt{\left(\frac{r^2 - x^2}{a^2 - x^2} \right)} \right] dx \\ &= 4/a \int_0^r \{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \} dx \\ &\quad + \frac{4b^2(a^2 - r^2)}{3a} \int_0^r \frac{x^2 dx}{\sqrt{\{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \}}} \\ &\quad + \frac{8ab^2(a^2 - r^2)}{3} \int_0^r \frac{x^2 dx}{(a^2 - x^2) \sqrt{\{ (r^2 - x^2) [a^2(b^2 - r^2) + (a^2 - b^2)x^2] \}}} \end{aligned}$$

$$\text{Let } x = r \cos \theta, \frac{r^2 (a^2 - b^2)}{b^2 (a^2 - r^2)} = e^2, \frac{r^2}{a^2 - r^2} = c.$$

$$\begin{aligned} V &= \frac{4br^2 \sqrt{a^2 - r^2}}{a} \int_0^{\frac{1}{2}\pi} (1 - e^2 \sin^2 \theta) \sin^2 \theta d\theta \\ &\quad + \frac{4br^2 \sqrt{a^2 - r^2}}{3a} \int_0^{\frac{1}{2}\pi} \frac{\cos^2 \theta d\theta}{\sqrt{1 - e^2 \sin^2 \theta}} \\ &\quad + \frac{8abr^2}{3 \sqrt{a^2 - r^2}} \int_0^{\frac{1}{2}\pi} \frac{\cos^2 \theta d\theta}{(1 + c \sin^2 \theta) \sqrt{1 - e^2 \sin^2 \theta}}. \\ \therefore V &= \frac{8abr^2(c+1)}{3c \sqrt{a^2 - r^2}} \Pi(e, c, \tfrac{1}{2}\pi) + \frac{8br^2(a^2 - r^2)}{3a} E(e, \tfrac{1}{2}\pi) \\ &\quad + \frac{4br^2}{3ce^2 \sqrt{a^2 - r^2}} [c(1 + e + e^2)(a^2 - r^2) - 2a^2 e^2] F(e, \tfrac{1}{2}\pi) \\ &= \frac{8a^3 b}{3 \sqrt{a^2 - r^2}} \Pi(e, c, \tfrac{1}{2}\pi) + \frac{8br^2(a^2 - r^2)}{3a} E(e, \tfrac{1}{2}\pi) \\ &\quad + \frac{4b \sqrt{a - r^2}}{3ae^2} [r^2(1 + c + e^2) - 2a^2 e^2] F(e, \tfrac{1}{2}\pi). \end{aligned}$$

Also solved by J. SCHEFFER.

MECHANICS.

87. Proposed by H. C. WHITAKER, M.E., Ph.D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

“He on his impious foes onward drove,
Drove them before him to the bounds
And crystal walls of Heaven; which opening wide
Rolled inward and a spacious gap disclosed
Into the wasteful deep; headlong themselves they threw
Down from the verge of Heaven.
Nine days they fell; Hell at last
Yawning received them whole and on them closed.”

Paradise Lost, Book VI.

Assuming Hell to be the center of the earth, and the only force acting on the lost spirits to be that of gravity due to the earth's attraction,—How far is Heaven?

II. Solution by the PROPOSER.

The published solution of No. 87, Mechanics, assumes (page 82, line 4) that $s = \frac{1}{2}gt^2$ is the formula for falling bodies for great distances.

Neglecting the change in passing from the surface to the center, the formula is $s^3 = \frac{gr^2 t^2}{2\pi^2}$. (See *Watson's Theoretical Astronomy*, page 46, eq. 28).